Pre Assignment Brazilian disc

A Brazilian disc is a circular disc loaded in 2 point diametric compression as shown in Figure 1. The disc has a complex two dimensional stress/strain field that is described by a closed form expression, i.e. there is an exact theoretical solution based on plane stress elasticity assumptions. In the experimental mechanics module you will carry out experiments on an aluminium disc of 80 mm diameter by 6 mm thick. You will extract stress and strain data from the disc experiments and you will use the theoretical solution to compare and discuss your experimental results.

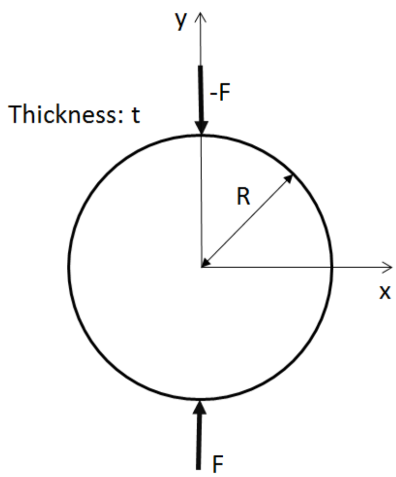

Figure 1 Brazilian disc notation

The disc is very useful as a test piece for experimental mechanics applications. It is used widely for validation, verification and calibration purposes because the experimental data can be compared with the theoretical solution. This problem has a closed form stress solution provided in [1]. The three in-plane stress components are:

$$\begin{cases} \sigma_{xx} = \dfrac{F}{\pi t R} - \dfrac{2F}{\pi t}\dfrac{(R-y)x^2}{\left[x^2+(R-y)^2\right]^2} - \dfrac{2F}{\pi t}\dfrac{(R+y)x^2}{\left[x^2+(R+y)^2\right]^2} \\ \sigma_{yy} = \dfrac{F}{\pi t R} - \dfrac{2F}{\pi t}\dfrac{(R-y)^3}{\left[x^2+(R-y)^2\right]^2} - \dfrac{2F}{\pi t}\dfrac{(R+y)^3}{\left[x^2+(R+y)^2\right]^2} \\ \sigma_{xy} = \dfrac{2F}{\pi t}\dfrac{(R-y)^2 x}{\left[x^2+(R-y)^2\right]^2} - \dfrac{2F}{\pi t}\dfrac{(R+y)^2 x}{\left[x^2+(R+y)^2\right]^2} \end{cases} \tag{1}$$

where σ_{xx} is the stress in the x direction, σ_{yy} is the stress in the y direction, σ_{xy} is the shear stress and t is the thickness of the disc.

In many cases the results from experiments are produced as strains. For a 2D plane stress elasticity case the stresses are related to the strains, ε, as follows:

$$\varepsilon_{xx} = \frac{1}{E}(\sigma_{xx} - \upsilon\sigma_{yy}) \tag{2}$$

$$\varepsilon_{yy} = \frac{1}{E}(\sigma_{yy} - \upsilon\sigma_{xx}) \tag{3}$$

$$\varepsilon_{xy} = \frac{(1+\nu)}{E}\sigma_{xy} \tag{4}$$

where E is Young's modulus and ν is Poisson's ratio.

The purpose of the assignment is to get familiar with the mechanics of the disc, understand better the nature of the assignments and be able to manipulate the above equations to obtain solutions that are comparable with the experimental data.

By using the above equations it is now possible to plot a map of the stresses and strains in the disc. This could be done using Matlab or a similar package to produce plots that are equivalent to those obtained with the experimental mechanics techniques. The stress field plots are shown in Figure 2. Here F = 9 kN.

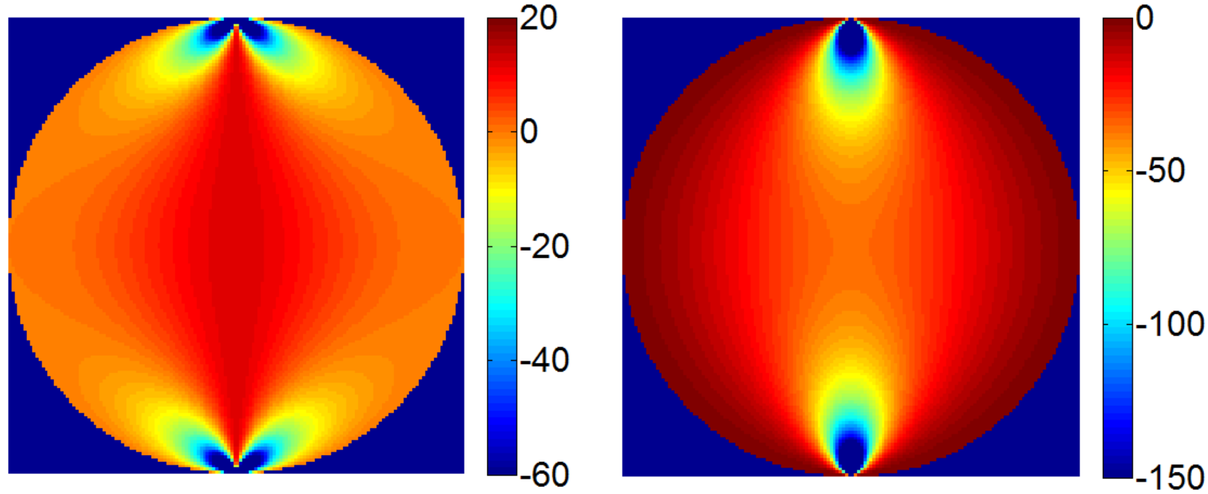

Figure 2 (a) x-direction stresses (MPa) (b) y-direction stresses (MPa)

The pre assignment is for you to produce contour plots using MATLAB similar to those shown in Figure 2, for the six in plane stress and strain components.

- Firstly note the dimensions of the disc used in the experiment (diameter = 80 mm and thickness = 6 mm)
- Select a suitable grid for your MATLAB programme (e.g. the disc diameter is 150 grid points).
- Apply the equations given in Equation (1) to the MATLAB grid (remember the centre of the disc is the origin (i.e. when $x = 0$ and $y = 0$).

- Produce three plots: σ_{xx}, σ_{yy} and σ_{xy}.
- Use E = 70 GPa and ν = 0.3 in conjunction with Equations (2 - 4) to compute the strains in the disc and hence 3 further plots of ε_{xx}, ε_{yy} and ε_{xy}.

In the work in assignment you will be asked to compare the theoretical solution with the results from each of the techniques. You will be asked to make a plot along the vertical centre line of the disc i.e. where $x = 0$ the solution is

$$\sigma_{xx} = \frac{F}{\pi t R} \tag{5}$$

$$\sigma_{yy} = \frac{F}{\pi t R}\left[1 - \frac{4R^2}{R^2 - y^2}\right] \tag{6}$$

1. Use equations (5) and (6) to verify your results along the vertical centre line.
2. Verify the stress free condition at ($x = R$, $y = 0$).
3. Derive an expression for the strains in the x and y direction along the vertical diameter of the disc.
4. Derive an expression for ($\sigma_{xx} + \sigma_{yy}$) along the vertical diameter of the disc.
5. Calculate the principal stresses (σ_1 and σ_2) at each point in the disc and plot as a contour plot. To do this you may wish to consult [2] for a revision of basic solid mechanics.

References

[1] S.P. Timoshenko and J.N. Goodier, Theory of elasticity, 3rd ed., McGraw-Hill New York, 567 p., 1970.
[2] D. W.A. Rees, The mechanics of solids and structures. (Chapter 13) McGraw-Hill Book Co Ltd., 1990.