

Infra-red thermography

Introduction

A body with a temperature above absolute zero will emit energy in the form of electromagnetic radiation/thermal radiation at its surface. As the temperature increases the quantity of heat transferred by means of thermal radiation increases. By using the amount of energy emitted in the form of electromagnetic radiation, accurate temperature measurements can be made over a wide range by means of infrared thermography. The spectral radiant emittance ($\Phi_{\lambda,b}$) for a blackbody in a hemisphere in the wavelength range from λ to $\lambda + \delta\lambda$ can be found using *Planck's law*, i.e.

$$\Phi_{\lambda,b} = \frac{C_1}{\lambda^5 (\exp \frac{C_2}{\lambda T} - 1)} \quad (1)$$

where C_1 is the first radiation constant $= 2\pi^2 h^3 c^2$ (h is Planck's constant, c is the speed of light and λ is wavelength), C_2 is the second radiation constant $= hc/k$ (k is the Boltzmann constant) and T is absolute temperature.

The spectral radiant emittance for a blackbody at various temperatures can be plotted against wavelength as shown in Figure 1.

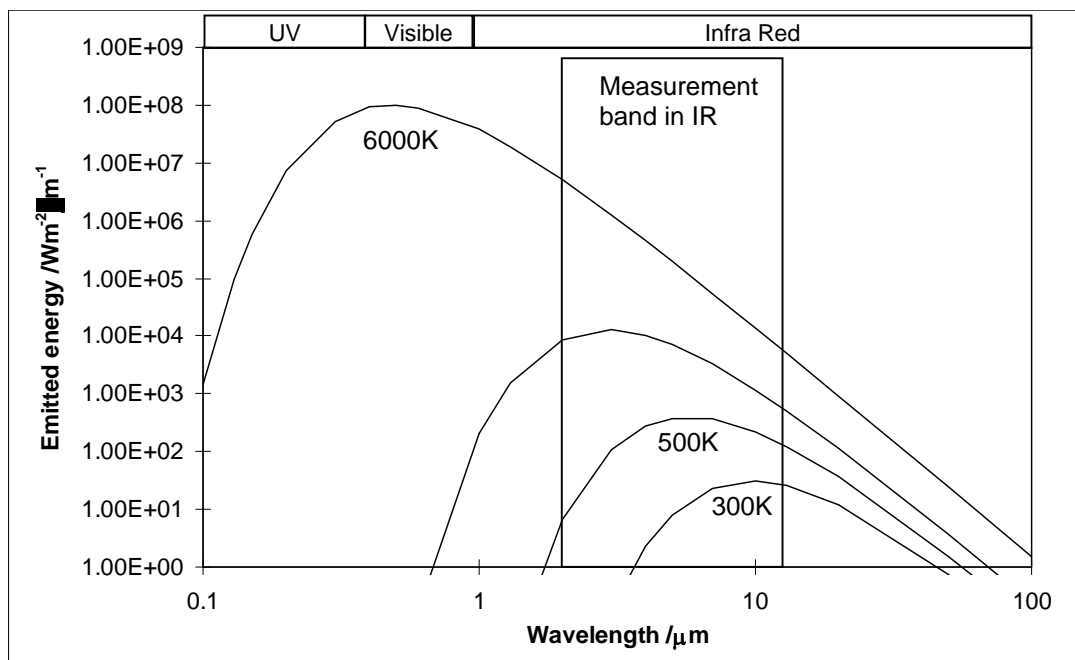


Figure 1: Radiant emitted energy from a black body according to Planck's Law

It is possible to detect the radiant emittance by using an infra-red detector. These are basically electronic devices that are sensitive to photon emission in the infra-red spectrum, which has a peak between 2 -12 μm at ambient as shown in Figure 1. In modern infra-red detection systems the detector is built into a camera system with a lens and electronics that enables the

camera to have a computer interface. Low sensitivity detectors are usually bolometers, higher sensitivity devices are used for strain measurement and these are usually *photon detectors*. The intensity of the photon collection defines the electrical signal output from the detector as the detector acts as a transducer turning a photon strike into a voltage signal. The purpose of this lecture is to understand the physics behind infra-red thermography before attempting to use thermoelastic stress analysis.

Stefan-Boltzman equation

It is possible to produce a finite integral to Planck's law (equation (1)) if the integral is considered between zero and infinity and this provides the well-known fourth-power *Stefan-Boltzmann* relationship for evaluating the radiant emittance over all wavelengths Φ_b as follows:

$$\Phi_b = 2\pi c^2 h \int_0^\infty \frac{d\lambda}{\lambda^5 (\exp(ch/kT\lambda) - 1)} \quad (2)$$

letting $x = ch/kT\lambda$ allows the integration to be processed by substitution with respect to x . Thus, x must be rewritten to make λ the subject and subsequently differentiated with respect to x . i.e.: $\lambda = \frac{ch}{kTx}$, $\frac{d\lambda}{dx} = \frac{ch}{kTx^2}$ and $\lambda^5 = \frac{c^5 h^5}{k^5 T^5 x^5}$.

Substituting the relationship for λ^5 and the derivative of λ with respect to x into equation (2) leaves:

$$\Phi_b = \frac{2\pi c^3 h^2 k^5 T^5}{c^5 h^5 k T} \int_0^\infty \frac{x^5 dx}{x^2 (\exp(ch/kT\lambda) - 1)} \quad (3)$$

and equation (3) can be simplified to:

$$\Phi_b = \frac{2\pi k^4 T^4}{c^2 h^3} \int_0^\infty \frac{x^3 dx}{\exp(x) - 1} \quad (4)$$

It is the next steps where the integral in equation (4), $I_s = \int_0^\infty \frac{x^3}{\exp(x) - 1} dx$, can be rewritten as a finite integral that allows equation (4) to be evaluated. The integral, I_s , can be expressed as:

$$I_s = \int_0^\infty \frac{x^s}{\exp(x) - 1} dx = \int_0^\infty x^s \exp(-x) \sum_{n=0}^\infty \exp(-nx) dx \quad (5)$$

where:

$$I_s = s! \sum_{n=1}^{\infty} \frac{1}{n^{s+1}} \quad (6)$$

for s equal to 3, as in this case, the integral may be rewritten using the Riemann Zeta function as:

$$I_3 = 6! \sum_{n=1}^{\infty} \frac{1}{n^4} = \frac{\pi^4}{15} \quad (7)$$

For other integrals of s , I_s may be written in the form: $I_s = s! \zeta(s+1)$ where $\zeta(x)$ is the Riemann Zeta function, which is available in mathematical tables for values of s . The values that are used to present the theoretical basis to the application of infra-red detectors to TSA are tabulated in Table 1.

s	I_s
2	2.4041
3	$\frac{\pi^4}{15}$

Using the integral in equation (4) the radiant emittance can be written as follows:

$$\Phi_b = \left(\frac{2\pi^5 k^4}{15c^2 h^3} \right) T^4 \quad (8)$$

This result can be simplified by rewriting the right hand side of equation (8) through the introduction of the *Stefan-Boltzmann* constant, B , as the bracketed term to give the well-known fourth-power *Stefan-Boltzmann* relationship for evaluating the radiant emittance over all wavelengths λ_b as follows:

$$\Phi_{\lambda,b} = BT^4 \quad (9)$$

It can be shown that $\Phi_{\lambda,b}$ has a maximum. The wavelength at which this occurs is defined as λ_{max} , and can be calculated from *Wien's displacement law*.

Photon detectors

The infra-red detectors used for strain measurement are photon detectors and not a bolometers. In a similar manner it is possible to obtain a discrete equation for the number of photons (N_b) emitted by an object at a specific temperature by dividing the energy in each wavelength interval by the energy carried by each photon. To evaluate the relationship for a general case the photon flux can be derived for the total number of photons per unit area and time by producing a closed form integral of the equation for spectral radiant emittance again this is only possible by considering the wavelength range between zero and infinity, as follows:

$$N_b = \int_0^{\infty} \frac{2\pi c}{\lambda^4 (\exp(\frac{hc}{\lambda kT}) - 1)} d\lambda \quad (10)$$

The integration is possible using the substitution method and the subsequent derivatives presented for equation (). Noting that here λ is raised to the power four in this case the λ terms can be rewritten for this derivation as:

$$\lambda^4 = \frac{c^4 h^4}{k^4 T^4 x^4} \quad (11)$$

Substituting into equation () leaves:

$$N_b = \frac{2\pi c k^4 T^4}{c^4 h^4} \frac{ch}{kT} \int_0^{\infty} \frac{x^4}{x^2 (\exp(x) - 1)} dx \quad (12)$$

which again can be simplified and rewritten to make use of the Riemann Zeta function:

$$N_b = \frac{2\pi k^3 T^3}{c^2 h^3} \int_0^{\infty} \frac{x^2 dx}{\exp(x) - 1} \quad (13)$$

The integral, in equation (13), may be evaluated in a similar manner to that used previously, where s is taken as two. Therefore the integrand may be expressed using the Riemann function for 2 as detailed in Table 1, incorporating the finite value for the integral into the equation leaves:

$$N_b = \frac{2\pi k^3 T^3}{c^2 h^3} (2.4041) \quad (14)$$

this formulation again allows the substitution of the *Stefan-Boltzmann* constant to provide the following relationship between the number of photons incident and the surface temperature:

$$N_b = \frac{0.370B}{k} T^3 \quad (15)$$

The quantity $0.370B/k = 1.52 \times 10^{15}$ photons $s^{-1} m^{-3} sr^{-1} K^{-3}$ and can be regarded as the *Stefan-Boltzmann constant for photodetectors*. Denoting the constant for photodetectors as B' , equation (15) simplifies to:

$$N_b = B' T^3 \quad (16)$$

The relationship presented in equation (16) shows that when considering the entire electromagnetic spectrum the total number of photons increases with the cube of the absolute temperature whilst the radiant emittance over the entire spectrum increases with the fourth-power of absolute temperature. The relationship for the number of photons (N_b) emitted from a

surface as derived above and presented in equation (16) has been used to relate the photon output from an infra-red detector to the temperature but these should be regarded as an approximation. For infra-red thermography there are two atmospheric windows of interest, one is located from 3 to 5 μm (most bolometers and photodetectors) and the other is from 8 to 12 μm (some photodetectors). The number of photons per unit area and time is obtained by integrating the over the operating wavelength band of the photodetector instead of the range 0 and ∞ . It is impossible to derive a closed form relationship, as presented for the radiant emittance or photon flux above, for practical narrow band IR detectors. Note that the Riemann Zeta function that enabled the integration is only valid for integrals over the 0 - ∞ range. Therefore it is necessary to consider other mathematical methods to express the relationship between the spectral radiant power and the absolute temperature between the wavelength limits of the infra-red detectors of interest. Clearly this will not yield the same result given by the formulation of equation (3) above. For detectors where the operating wavelength is much less than λ_{max} then the response of the detector to temperature changes follows an approximate power law, i.e. $N_b \propto T^n$. Therefore it is possible to propose an equation that relates the surface temperature of a body to the total number of photons emitted over a particular wavelength range as follows

$$N_{b_\lambda} = B' T^n \quad (17)$$

where B' is a constant that is dependent on the detector. The index n can be evaluated by determining N_{b_λ} from equation (5) by numerical integration over the wavelength of interest for a variety of temperatures. Taking logs of equation (6) will yield a simple linear form, i.e.

$$\ln N_{b_\lambda} = \ln B' + n \ln T \quad (18)$$

and n can be determined from the slope of a plot of $\ln N_{b_\lambda}$ against $\ln T$; B can be derived from the intercept of the plot.

Equations (9) and (18) can be used as a basis for calibration, however most instruments are supplied with empirical calibration curves from the manufacturer.

Specimen preparation

When radiation impinges on a body it is either transmitted through the body, absorbed by the body or reflected away from the body, so that

$$\tau + a + r = 1 \quad (19)$$

where τ is the transmissibility, a is the absorptivity and r is the reflectivity.

Engineering materials are usually opaque in the infra-red region, even if they are transparent to visible light, e.g. glass. The transmitted energy is therefore zero. So equation (19) can be re-written as follows:

$$a + r = 1 \tag{20}$$

The reflectivity is then $1 - a$, so a portion of incident radiation is reflected back to the detector. Care must be taken to avoid anomalous readings from reflections from heaters and the sun. The absorbed energy is therefore equal to the emitted energy so

$$\alpha = e \tag{21}$$

where e is the emissivity.

Table 2 lists common engineering materials and their emissivities. The variability in emissivity means that care is required to interpret the measured temperature, and to take account of background radiation. Often a layer of matt black paint is applied to the surface of a material to create an enhanced and uniform emissivity. It is interesting that this is not usually necessary for polymeric materials.

Table 2: Typical emissivity values		
Material		Typical ϵ
Copper	polished	0.02-0.05
	oxidised	0.60-0.78
Aluminium	polished	0.04-0.05
	oxidised	0.10-0.31
	anodised	0.55-0.72
Steel	polished	0.07-0.08
	oxidised	0.79
	rolled sheet	0.60
Paint	white enamel	0.92
	green enamel	0.82
	matt black lacquer	0.97
	white lacquer	0.80-0.92
	aluminium	0.27-0.60
	oil	0.89-0.97
PVC		0.91-0.93
Plastic, general	black	0.95
	white	0.84
Insulating tape	black	0.97

Sources: AGEMA; CIBSE Guide C; ASHRAE

Infra-red systems are usually calibrated using a black body therefore it is necessary to input a surface emissivity prior to calibrating the response into a temperature value.