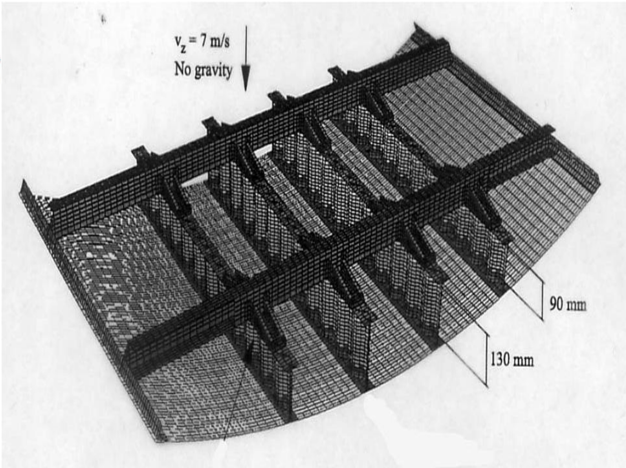
Identification of a delay damage mesomodel for localization and rupture of composites: Identification strategy

Pierre Feissel[a] Olivier Allix[a] Pascal Thévenet[b]

[a]*LMT, ENS Cachan*

with the support of: [b]*EADS-CCR*

Industrial context



- **The Aircraft industry develops**

  composite crash absorbers

  the tests are still essential

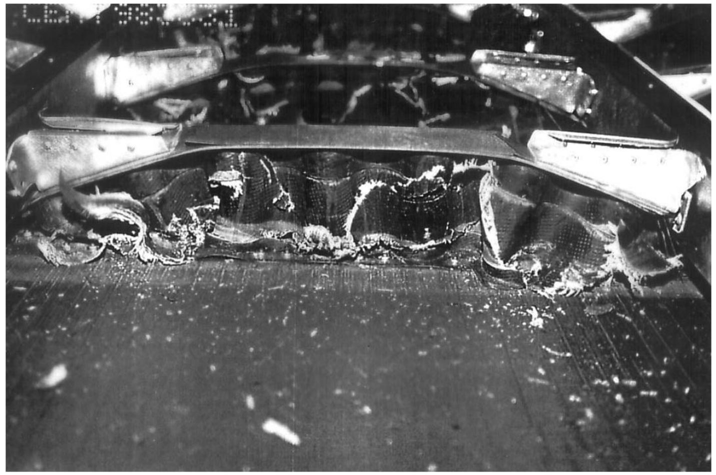
  (quite costly)

  ↪ **strong will to develop numerical tools**

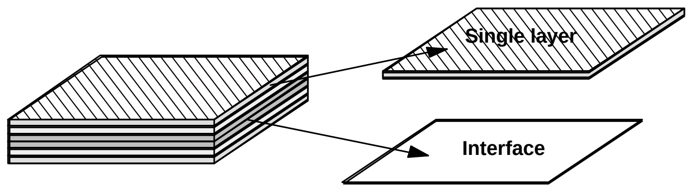
Simulations predict the energy absorption ⟹ models taking into account :

- ⇝ **the nature of the behavior**
- ⇝ *the rate effects*

  [*Harding, Coutellier, Baptiste,...*]
- ⇝ **the post-peak behavior**
- ⇝ *the fragmentation*



Delay damage mesomodel



[*Ladevèze,86*] [*Deü, Allix,97*]

■ **State law:**

$$\sigma = E.(1-d).\varepsilon$$

$$Y = \frac{E.\varepsilon^2}{2}$$

■ **Evolution law:**

$$\dot{d} = \frac{1}{\tau_c}.\left(1 - e^{-a.\langle f(Y)-d\rangle_+}\right)$$

with $d = f(Y)$ the static law

for example: $f(Y) = \frac{\sqrt{Y}}{\sqrt{Y_c}}$

■ **Mesomodel:**

$\hookrightarrow$ good description of the phenomena, identified from standard tests

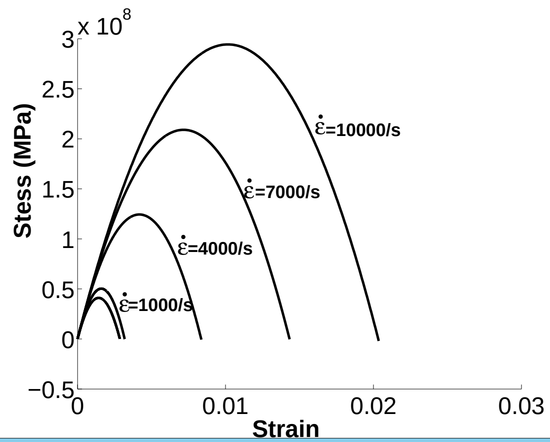
■ **Introduction of a delay effect:** to deal with rupture and localization

◊ It is not a classic rate effect

◊ Small $\dot{d} \Rightarrow \quad d = f(Y)$, the static law

◊ The damage rate is bounded

$$\dot{d} \in \left[0, \frac{1}{\tau_c}\right]$$



? How to identify the delay parameters?

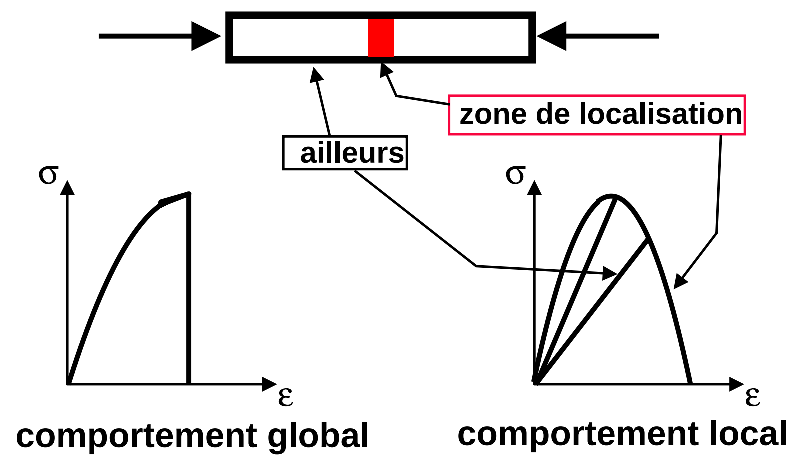
■ **From tests where rupture occurs:**

↪ localization phenomenum.

⟹ **strongly heterogenous tests**

↪ **strong uncertainties**

on the boundary conditions



⇝ boundary conditions known by their mean.

(*example*: structural tests)

⇝ distance between the measurement points and the specimen's boundary

⇝ difficulties in tests analysis (rupture in dynamics)

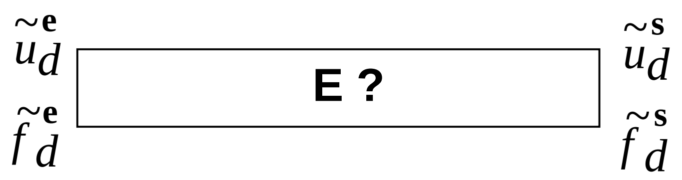
<u>Aim of the work</u>:

to construct a robust identification strategy in this context

Identification framework

? How behave identification methods in the case of strong scattering of measurements ?

↪ Study on an example : identification of the Young's modulus of a beam.



measurements of forces and displacement at both ends of the beam.

the direct problem is ill-posed (in most cases, there is no solution) too much datas

↪ quantification of the compatibility between the boundary conditions and the material parameters.

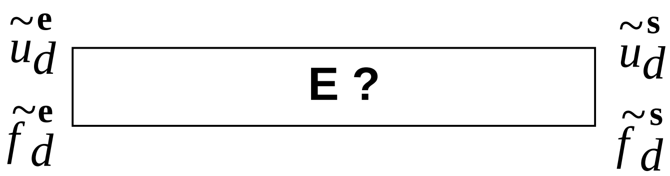
- **Inverse approach :** two steps.
  - ⇝ *First step:* define a calculation from the experimental datas for a given *E*
  - ⇝ *Second step:* evaluate the quality of *E* through a functional of the solution fields of the calculation, and minimize it for identification.

Method proposed by *Rota*

- **In order to recover a well-posed problem,**

some equations have to be released:

↪ Split into two auxiliary problems [*Rota*]



Prescribed displacements

↪ yields a solution field: $u_{CA}(E)$

Prescribed forces

↪ yields a solution field: $u_{SA}(E)$

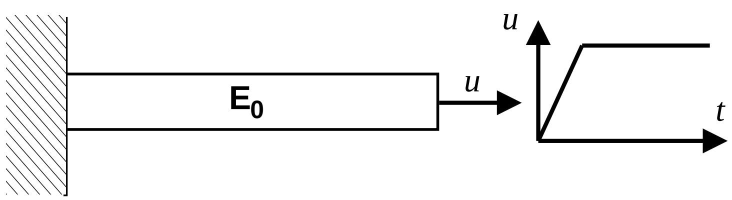
definition of a distance between the two calculations : $e(u_{CA}(E), u_{SA}(E))$

- **The identification becomes :**

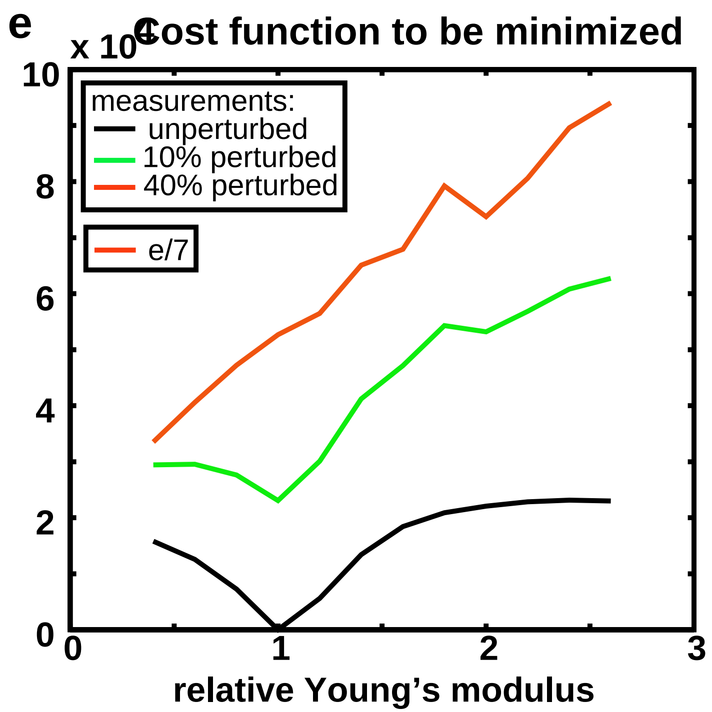
$$\min_E C(E) = \min_E e(u_{CA}(E), u_{SA}(E))$$

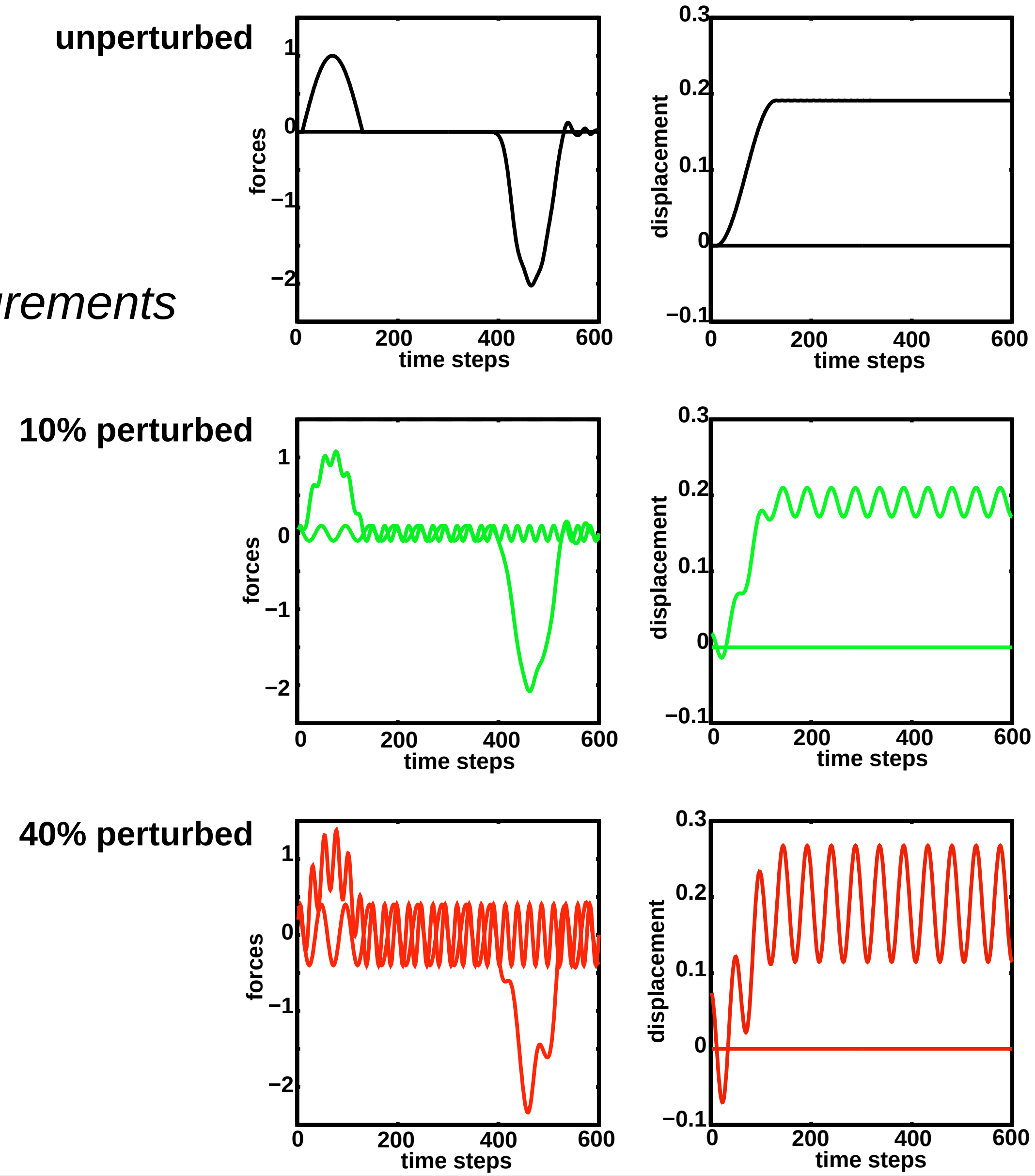
Identification on an example

Numerical test



↪ *Creation of perturbed mesurements*





Taking into account the uncertainties

Remarks on the previous method :

◊ There are multiple ways to split the experimental information.

◊ The experimental measurements are strongly prescribed to the calculations.

$\hookrightarrow$ Use of the errror in constitutive relation principles
inspired by what is done in model updating in vibration.
[*Ladevèze, Deraemaeker*]

■ **Spliting of the quantities into two groups:**

Reliable	Uncertain
Equilibrium: $\rho.\ddot{u} - div\,\sigma = 0$	Constitutive relation: $\sigma = E.\epsilon$ Measurements: $\tilde{u}_d$ and $\tilde{f}_d$

exactly verified — **verified at best**

$\tilde{u}_d^e$, $\tilde{f}_d^e$ — **E ?** — $\tilde{u}_d^s$, $\tilde{f}_d^s$

Identification process

■ **First step: definition of the basic problem**, for a given E

(confronting the measurements and the model)

solving the mechanics ill-posed problem

Find *the fields* u, σ, u_d, f_d *minimizing:*

$$J(\sigma, u, u_d, f_d) = \int_0^T \frac{1}{2} \int_\Omega (\sigma - E.\epsilon)\, .E^{-1}. (\sigma - E.\epsilon) + \int_{\partial\Omega_f} d_f(f_d, \tilde{f}_d) + \int_{\partial\Omega_u} d_u(u_d, \tilde{u}_d)$$

under the constraints:

$$u \text{ CA à } u_d, \quad \sigma \text{ DA à } f_d, \quad \rho.\ddot{u} + div\, \sigma = 0$$

$\hookrightarrow$ yields the solution fields: $\sigma(E), u(E), u_d(E), f_d(E)$

■ **Second step : identification of *E*** *(estimation of the quality of E)*

$$\min_E g(E) = \min_E J(\sigma(E), u(E), u_d(E), f_d(E))$$

Modified error in constitutive relation

The functional to be minimized is:

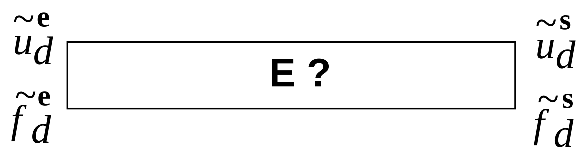
$$J(\sigma,u,u_d,f_d)=\int_0^T \frac{1}{2}\underbrace{\int_\Omega (\sigma-E.\epsilon).E^{-1}.(\sigma-E.\epsilon)}_{\text{error in constitutive relation}}+\underbrace{\int_{\partial\Omega_f} d_f(f_d,\tilde{f}_d)+\int_{\partial\Omega_u} d_u(u_d,\tilde{u}_d)}_{\text{distance between the measurements and the boundary conditions}}$$

error in constitutive relation

distance between the measurements and the boundary conditions

Remark :

If the measurements correspond to the BC for the good Young's modulus, the formulation gives the solution fields of the problem in forces or displacement.



One has to:

- Solve the basic problem
- Minimize the cost function to identify E.

Solving of the basic problem

■ **Solution of the basic problem:**

◊ the minimization under constraint is solved

by introducing the Lagrangian:

$$L = J(\sigma, u, u_d, f_d) - \int_0^T \left\{ \int_{\partial_u \Omega} (u - u_d) . \lambda - \int_\Omega (\rho . \ddot{u} - div(\sigma)) . u^* + \int_{\partial_f \Omega} (f_d - \sigma . n) . u^* \right\}$$

◊ the minimum of J under constraint amounts to the stationnarity of L:

$$\delta L = 0$$

↪ the solution must then verify a space-time differential system, with:

- some initial conditions for u: $u(0) = u_0$ et $\dot{u}(0) = \dot{u}_0$
- some final conditions for u^*: $u^*(T) = 0$ et $\dot{u}^*(T) = 0$

↪ *Need to develop some adapted solving methods*

■ $\sigma = E.\varepsilon(u + u^*)$ **so** u^* **measures the error in constitutive relation.**

Numerical solving of the differential system

■ **Continous system on $]0, L[$:**

$$\begin{cases} \sigma = E.\left(u_{,x} + u^*_{,x}\right) \\ \rho.\ddot{u} - E.u_{,xx} - E.u^*_{,xx} = 0 \\ \rho.\ddot{u}^* - E.u^*_{,xx} = 0 \end{cases}$$

■ **Boundary conditions:**

$$\begin{cases} \left[E.\left(u_{,x} + u^*_{,x}\right) - \frac{1}{B}u^*\right]_0^L = \left[\tilde{f}_d\right]_0^L \\ \left[u - \frac{E}{A}u^*_{,x}\right]_0^L = \left[\tilde{u}_d\right]_0^L \end{cases}$$

■ **Initial and final conditions:**

$u(x,0) = u_0$ and $\dot{u}(x,0) = \dot{u}_0$

$u^*(x,T) = u^*_T$ and $\dot{u}^*(x,T) = \dot{u}^*_T$

$\hookrightarrow$ Finite elements formulation and choice of a temporal scheme

Relationship between the nodal unknowns at the step n and the step $n+1$:

$$\begin{bmatrix} U_{n+1} \\ \dot{U}_{n+1} \\ U^*_{n+1} \\ \dot{U}^*_{n+1} \end{bmatrix} = A_n . \begin{bmatrix} U_n \\ \dot{U}_n \\ U^*_n \\ \dot{U}^*_n \end{bmatrix} + B_n$$

with U_0, $\dot{U}_0$, $U^*_{N_t}$, $\dot{U}^*_{N_t}$ prescribed.

$\hookrightarrow$ U^*_0, $\dot{U}^*_0$ are needed for an incremental solving

Numerical solving of the differential system

Thanks to the previous relationship, the *IC* can be related to the *FC* :

$$\begin{bmatrix} U_{N_t} \\ \dot{U}_{N_t} \\ U^*_{N_t} \\ \dot{U}^*_{N_t} \end{bmatrix} = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix} \cdot \begin{bmatrix} U_0 \\ \dot{U}_0 \\ U^*_0 \\ \dot{U}^*_0 \end{bmatrix} + \begin{bmatrix} T_1 \\ T_2 \end{bmatrix}$$

$$\Longrightarrow \quad \begin{bmatrix} U^*_0 \\ \dot{U}^*_0 \end{bmatrix} = {M_{22}}^{-1} . \left(\begin{bmatrix} U^*_{N_t} \\ \dot{U}^*_{N_t} \end{bmatrix} - M_{21} . \begin{bmatrix} U_0 \\ \dot{U}_0 \end{bmatrix} - T_2 \right)$$

$\hookrightarrow$ With these *IC*, a direct calculation can be done and the system is solved.

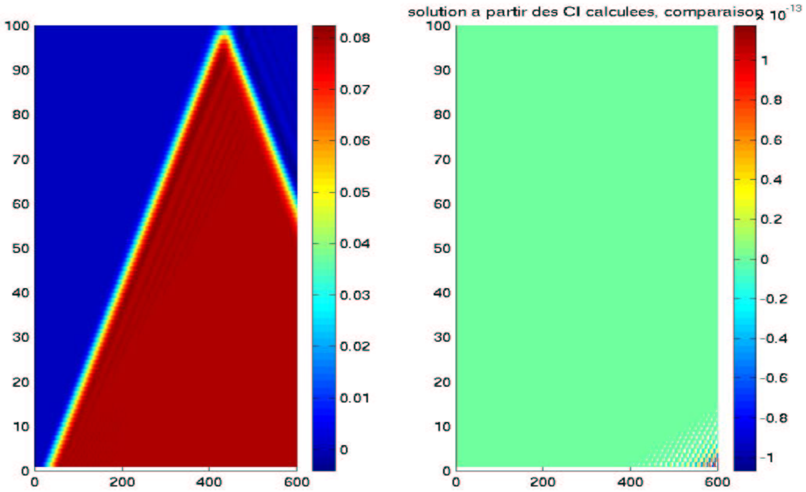
$\leadsto$ Other (more robust) methods do exist

$\leadsto$ This method can be extended to the non-linear case

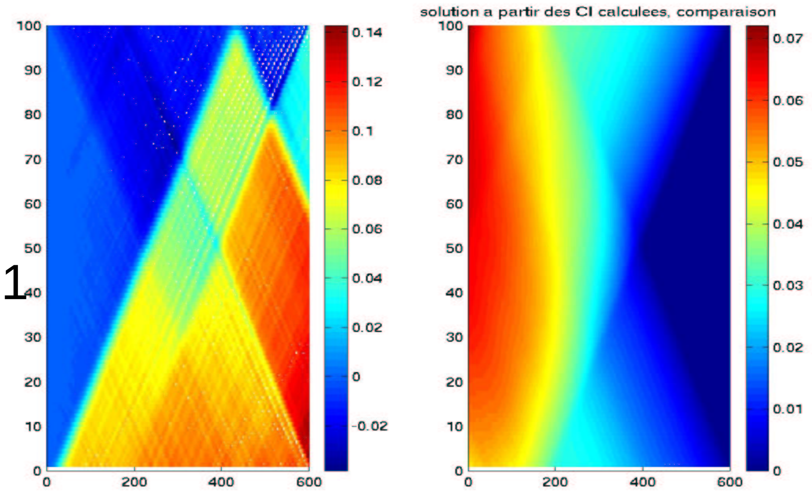
(iterative method using the gradient of *FC*/*IC*)

Solution fields on an example

u u^* u u^*

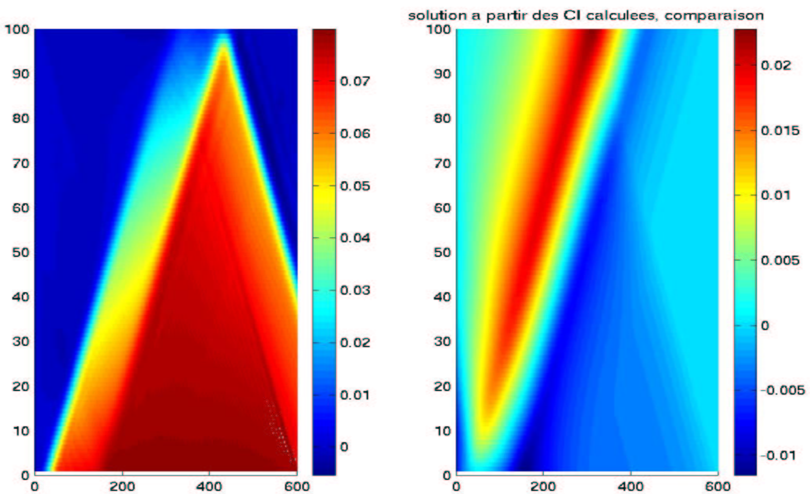


$$\frac{E}{E_0} = 1$$

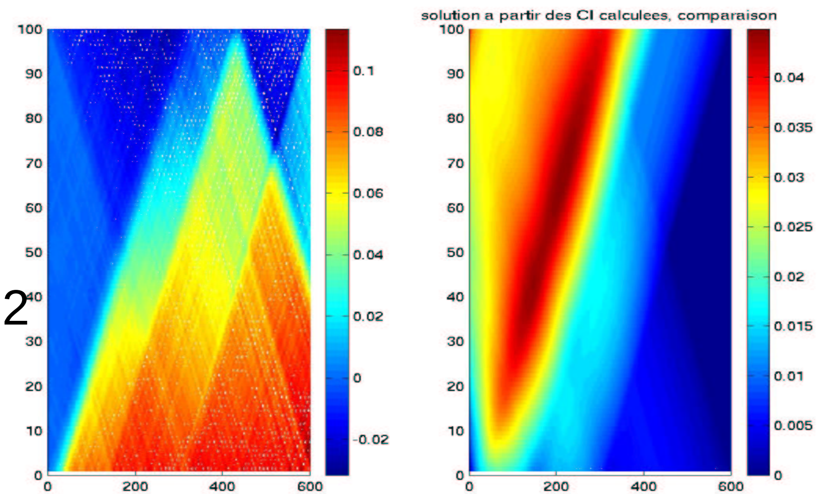


unperturbed measurements

perturbed measurements



$$\frac{E}{E_0} = 2$$



Identification : gradient's evaluation

- **Second step:** identification of E

$$\min_E g(E) = \min_E J(\sigma(E), u(E), u_d(E), f_d(E)) \Rightarrow \quad \text{minimization strategy}$$

- **Noting that:** $L(\sigma(E), u(E), u_d(E), f_d(E), u^*(E), \lambda(E), E) = g(E)$

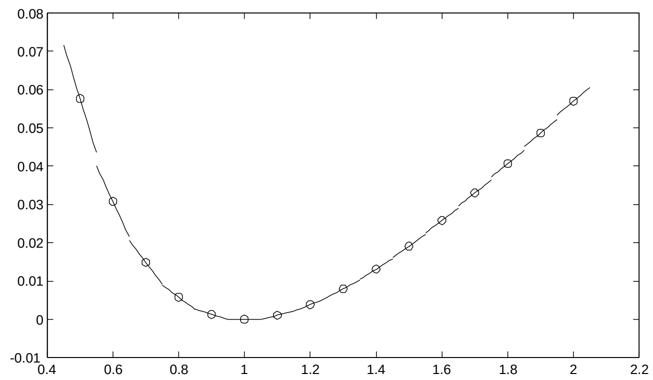
  one has:

$$Dg(E).q = \underbrace{\frac{\partial L}{\partial \sigma}\cdot\frac{\partial \sigma}{\partial E}.q + \frac{\partial L}{\partial u}\cdot\frac{\partial u}{\partial E}.q + \frac{\partial L}{\partial u^*}\cdot\frac{\partial u^*}{\partial E}.q + \frac{\partial L}{\partial u_d}\cdot\frac{\partial u_d}{\partial E}.q + \frac{\partial L}{\partial f_d}\cdot\frac{\partial f_d}{\partial E}.q}_{=0} + \frac{\partial L}{\partial E}.q$$

since $(\sigma(E), u(E), u_d(E), f_d(E), u^*(E), \lambda(E))$ are the solution fields of the basic problem.

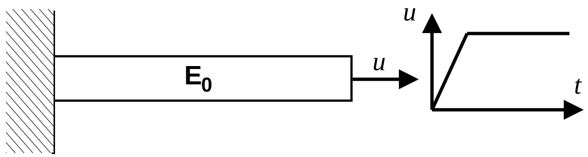
- **Computation of the gradient from the solution fields:**

$$Dg(E).q = \frac{\partial L}{\partial E}.q$$



Methods comparison

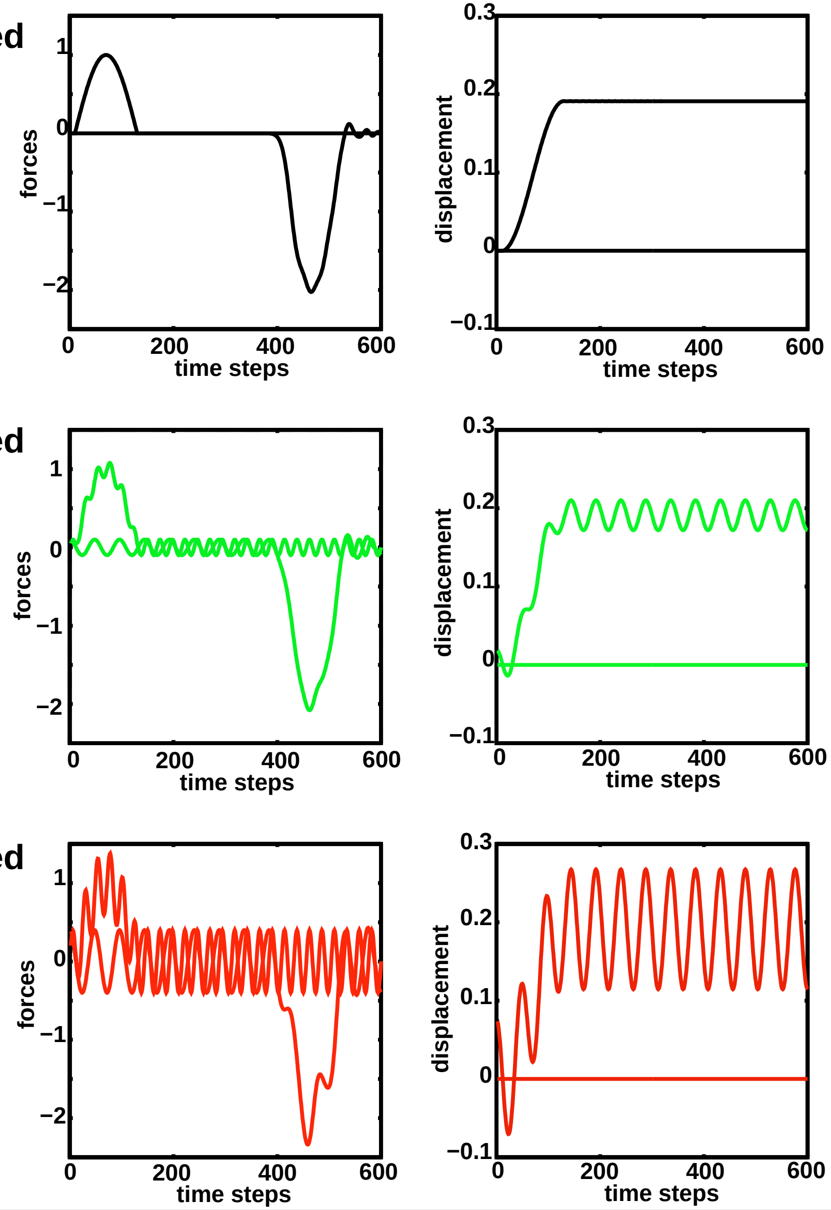
numerical test



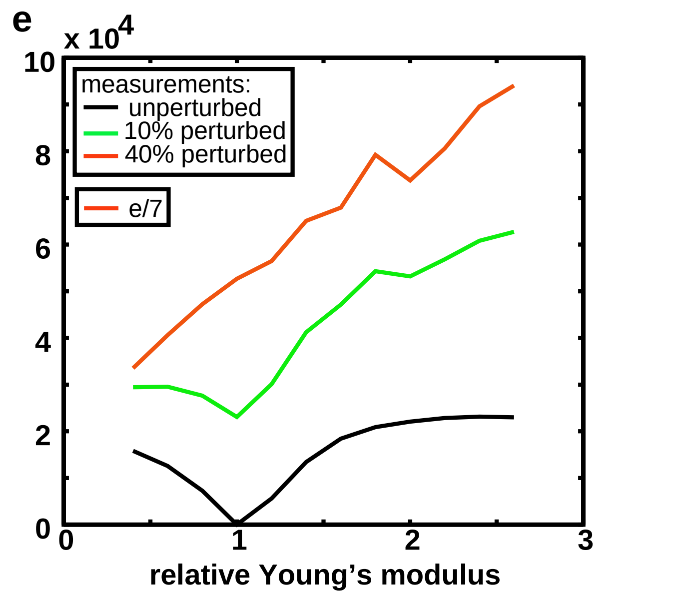
unperturbed

10% perturbed

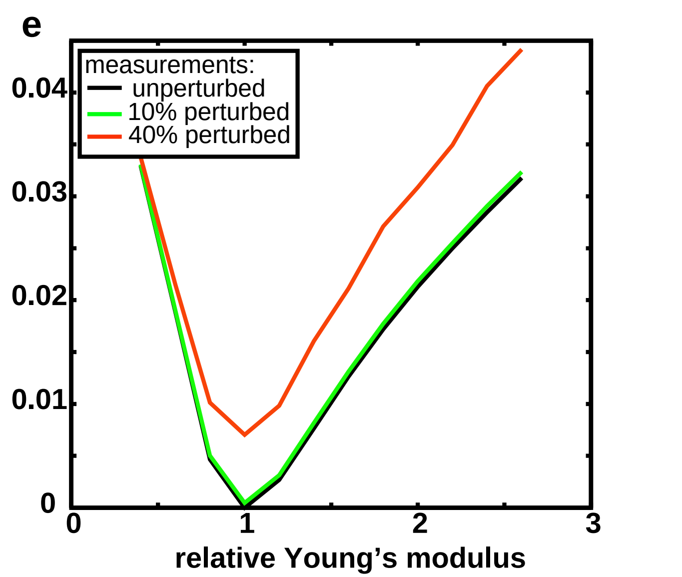
40% perturbed



[Rota]

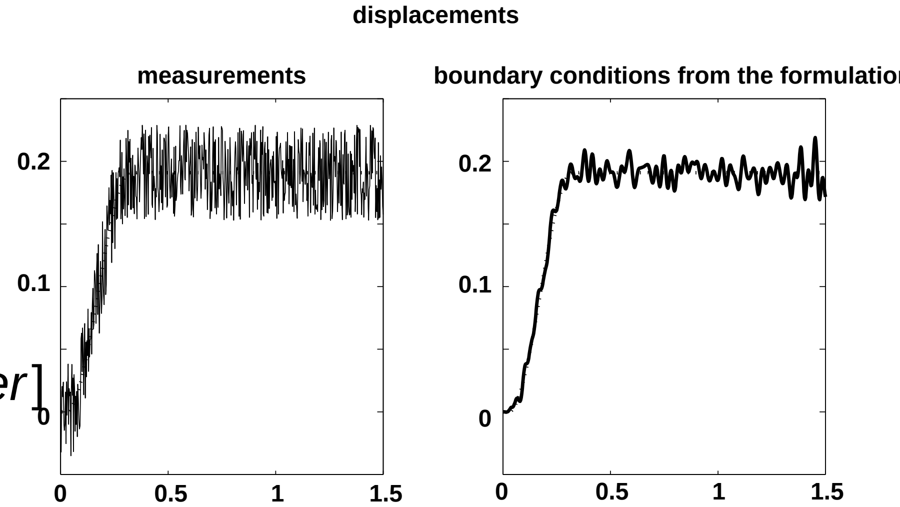


eRdC



Influence of the distance to the measurements

$$J(\sigma, u, u_d, f_d) = \int_0^T \frac{1}{2} \underbrace{\int_\Omega (\sigma - E.\epsilon).E^{-1}.(\sigma - E.\epsilon)} + \underbrace{\int_{\partial\Omega_f} d_f(f_d, \tilde{f}_d)} + \underbrace{\int_{\partial\Omega_u} d_u(u_d, \tilde{u}_d)}$$



■ **It can be seen as a regularization term:**

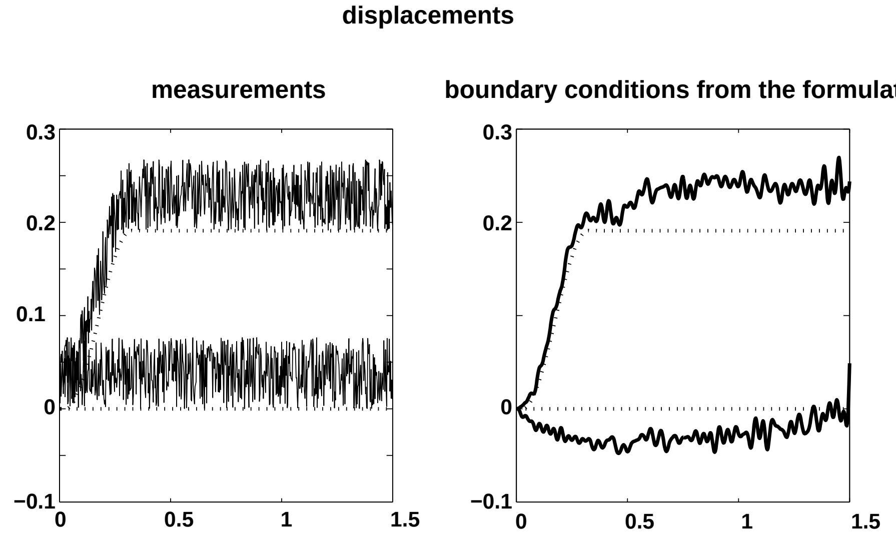
◊ How to choose the weight of this term ?

[*Philips-Tikhonov, Deraemaeker*]

$\hookrightarrow$ the terms are adimensioned.

◊ Is it working without it?

◊ Shall we keep it for the identification step?



Formulation without the term of distance to the measurements:

(Experimental boundary conditions strongly prescribed)

■ First step: definition of the basic problem

Find *the fields* u, σ *minimizing:*

$$J_2(\sigma, u) = \int_0^T \frac{1}{2} \int_0^L (\sigma - E.u_{,x}) .E^{-1}. (\sigma - E.u_{,x})$$

under the constraints:

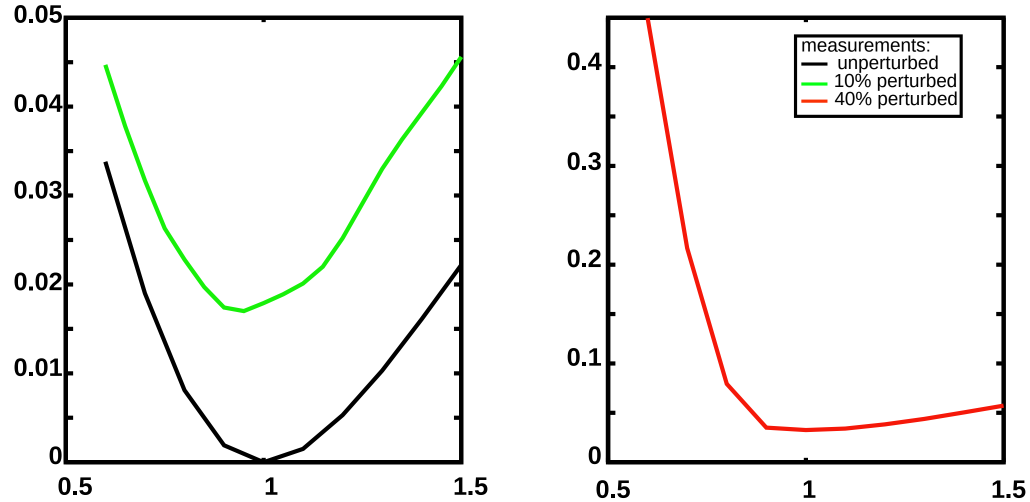
u CA à $\tilde{u}_d$, σ DA à $\tilde{f}_d$, $\rho.\ddot{u} + div\,\sigma = 0$

$\hookrightarrow$ yields the solution fields: $\sigma(E), u(E)$

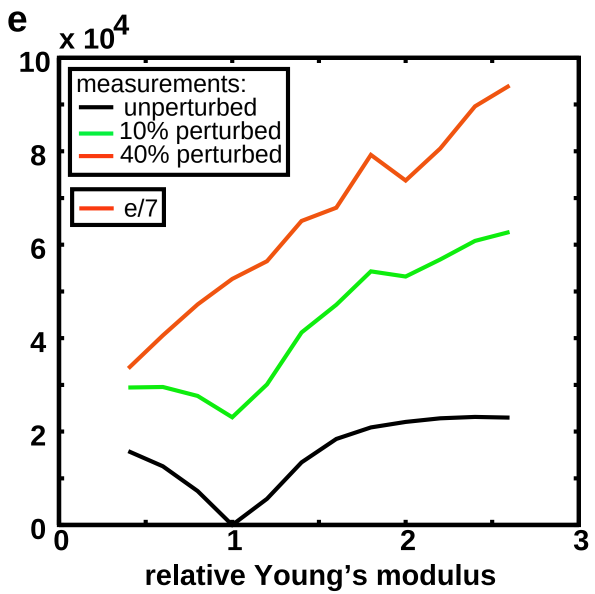
■ Second step : identification of *E* *(estimation of the quality of E)*

$$\min_E g_2(E) = \min_E J_2(\sigma(E), u(E))$$

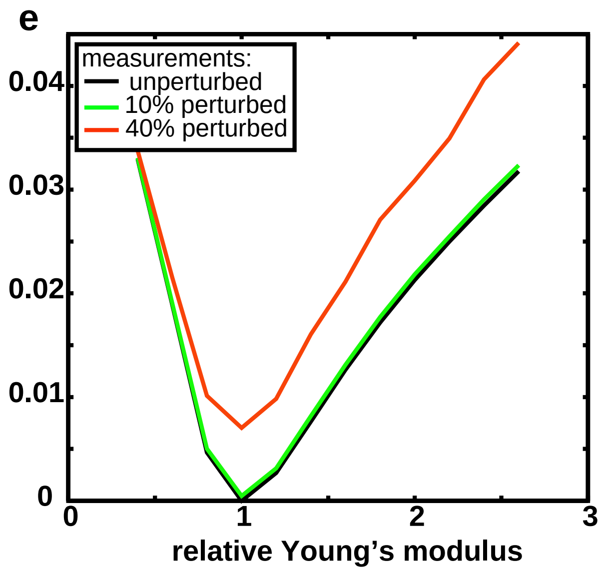
? Is it working without it?



unmodified error in constitutive relation (Strong *BC*)



[*Rota*]



EiCR with the distance to the measurements

? Shall we keep it for the identification step?

■ **The choice of the cost function is independant from the basic problem**

◊ Basic problem: (with the regularization term)

Trouver *les champs* u, σ, u_d, f_d *minimisant :*

$$J(\sigma, u, u_d, f_d) = \int_0^T \frac{1}{2}\int_\Omega (\sigma - E.\epsilon)\,.E^{-1}.\,(\sigma - E.\epsilon) + \int_{\partial\Omega_f} d_f(f_d, \tilde{f}_d) + \int_{\partial\Omega_u} d_u(u_d, \tilde{u}_d)$$

sous les contraintes:

$$u \text{ CA à } u_d, \quad \sigma \text{ DA à } f_d, \quad \rho.\ddot{u} + div\,\sigma = 0$$

$\hookrightarrow$ yields the **regularized** solution fields: $\sigma(E), u(E), u_d(E), f_d(E)$

◊ Choice for the cost function :

$$\min_E g(E) = \min_E J(\sigma(E), u(E), u_d(E), f_d(E))$$

OR

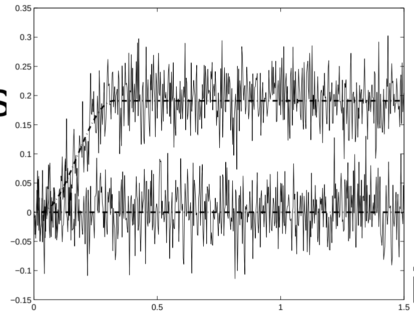
$$\min_E g_2(E) = \min_E J_2(\sigma(E), u(E)) = \min_E \int_0^T \frac{1}{2}\int_0^L (\sigma - E.u_{,x})\,.E^{-1}.\,(\sigma - E.u_{,x})$$

? Shall we keep it for the identification step?

Gaussien noise realization

to perturbe the measurements

($\sigma_0 = 0,2 \,.\, u_{max}$ or f_{max})



↪ for each realization, the … is identified for the two cost function:

1st case : weight = 1/magnitude of each term

cost function with	2 terms	1 term
mean	$0,987$	0.997
variance	$6,7.10^{-4}$	$7,3.10^{-5}$

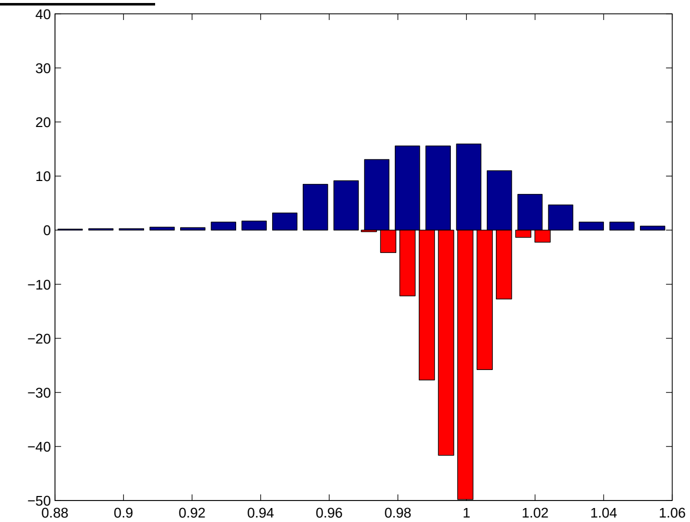
2d case : weight/10 for the distance

to the measurements.

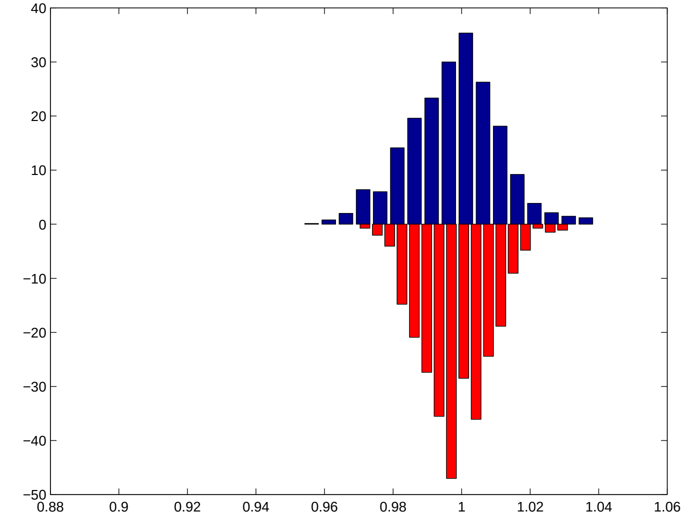
(changes the two methods)

cost function with	2 terms	1 term
mean	$0,998$	0.999
variance	$1,69.10^{-4}$	$9,34.10^{-5}$

1st case :



2d case :



Conclusion about the distance to the measurements

- **In the case of strong scattering** of the measurements,
  the term of distance is useful to allow identification.

- **The choice of the weight** in front of this term depends on the level of perturbation

- **Without information** on the level of uncertainties,
  the identification step without the term of distance seems robust

Delay effect identification

$\tilde{u}_d^e$, $\tilde{f}_d^e$ → **τ_{c},a ?** → $\tilde{u}_d^s$, $\tilde{f}_d^s$

■ Spliting of the quantities into two new groups:

Reliable	*Uncertain*
Equilibrium: $\rho.\ddot{u} - div\sigma = 0$	Evolution law: $\dot{d} = \frac{1}{\tau_c}.\left(1 - e^{-a.\langle f(Y)-d\rangle_+}\right)$
State law: $\sigma = E.(1-d).\epsilon$ $Y = \frac{E.\epsilon^2}{2}$	Mesurments: $\tilde{u}_d$ et $\tilde{f}_d$

$\hookrightarrow$ definition of a new basic problem taking this spliting into account.

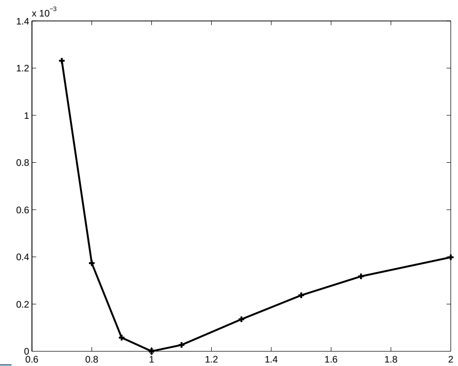
***Find** the fields u, σ, d, u_d, f_d minimizing:*

$$J(\sigma, u, u_d, f_d) = \int_0^T \int_\Omega \eta_\varphi(\dot{d}, Y; d) + \int_{\partial\Omega_f} d_f(f_d, \tilde{f}_d) + \int_{\partial\Omega_u} d_u(u_d, \tilde{u}_d)$$

under the constraints:

$$u \text{ CA à } u_d, \quad \sigma \text{ DA à } f_d, \quad \rho.\ddot{u} + div\, \sigma = 0, \quad \sigma = E.(1-d).\varepsilon$$

First results on a 0*D* example are promising:



Conclusion and outlook

■ **A first step in order to build a robust identification** method for problems with very imprecise boundary conditions such as those encountered in crash tests

■ **Present work concerns the case of damage with localization** and especially the resolution of the coupled direct-retrograde non-linear wave problem

-> first results are promising

■ **Experiments** are currently done by EADS and ENSAM Paris